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THE DOCTRINE OF SPACE AND TIME.

III. THE BERKELEIAN DOCTRINE OF SPACE.

IT is clear from what was said in my last paper that the Kantian doctrine is a house divided against itself, and that, unless we elect to embrace the motto: *credo quia absurdum est*—a motto not now in fashion in most departments of human knowledge—we are under obligations either to modify it or to repudiate it altogether.

What shall we do? Shall we maintain that space is *not* infinitely divisible? If we have the temerity to do this, we shall find drawn up against us, not merely the philosophers, but with them a formidable array of those who, like Clifford, care not a doit for philosophers, but hold very definite notions regarding points, lines, surfaces, and solids, and express these opinions with much emphasis. The mathematician usually takes little interest in such distinctions as that between 'intuition' and 'conception'; but he insists strenuously that it is absurd to maintain that a surface may be so narrow that, when split longitudinally, it is divided into two lines; or a line so short that, when bisected, it yields only a brace of points. Mathematics, he affirms, can recognize no such lines or surfaces.

And in this the mathematician is entirely in the right. The space with which he is concerned *is* infinitely divisible; his solids do not split up into surfaces, his surfaces into lines, and his lines into points. But, then, he is not dealing with a space immediately given in intuition; he is dealing with *real* space. He has passed from sign to thing signified, without remarking the distinction between them, and though this distinction may not greatly concern him when he remains on his own ground, it is one of the utmost moment to the metaphysician. Indeed, it is just the failure to recognize it that has introduced into the Kantian doctrine the inconsistencies previously discussed. That doctrine is so near to the truth that it needs but a little modification to make it quite satisfactory. This I must try to make clear.

We have seen that Kant held that every object of intuition must consist of part out of part, whether we can prove it to be so constituted or not. "All intuitions," he maintains elsewhere in the *Critique*,[1] "are extensive quantities." "By an extensive quantity," he explains, "I mean one in which the representation of the parts makes possible the representation of the whole (and hence, necessarily antecedes this). I cannot represent to myself any line, however small, without drawing it in thought, *i. e.*, from a point generating all its parts successively, and thus alone producing the intuition. So it is also in the case of every, even the smallest, portion of time. In it I represent to myself only the successive progress from moment to moment, and this by the addition of all the bits of time (*Zeittheile*), finally begets a determinate quantity of time. Since the pure intuition in all phenomena is either of space or of time, every phenomenon, as intuition, is an extensive quantity, for it can only be cognized in apprehension through the addition of part to part. Hence all phenomena are intuited as aggregates, as consisting of a multiplicity of previously given parts. This is not the case with qùantities of every description, but only with those that are represented and apprehended by us as in their nature extensive quantities."

The reader of the second of this series of papers will find in this passage a good deal to object to. To represent to myself any line, however small, I must produce it bit by bit; I must successively add *all* its parts. How many of these parts are there? An endless number. And are these bits of line ready to hand, or must they be produced 'from a point'? And what is meant by a "successive progress from moment to moment"? Are moments indivisible, or are they bits of time? Evidently the latter. They, in turn, then, are a problem, and must be obtained as the result of an endless addition of parts. The successive addition of portions of space and of time seems simple only when one forgets for the moment that one is a Kantian. That is what Kant has done here; he makes space and time out of spaces and times; but he leaves us wholly in the dark as to how those bits of space and time that we are to piece together come into being.

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[1] *Critique of Pure Reason*, Transcendental Logic, Axioms of Intuition.

There is a leap from a point — and they somehow appear ; the rest is simple. But we must not ask how we 'drew' the first bit of line, or how we 'begat' a moment. Moreover, if *all* phenomena are "cognized in apprehension through the addition of part to part," or "intuited as aggregates," how about the *minimum sensibile*, which is inferred to have parts, although we cannot perceive it to be composed of such ? Do we 'intuit' this as an aggregate, even while it seems to us to be simple ?

But I must not dwell upon these inconsistencies, for they have been sufficiently discussed already. In the division of the *Critique* from which I have just been quoting, Kant again makes it evident that he is led to take the unfortunate position that he does take, by the supposed necessity of avoiding a clash with mathematical doctrine. "Empirical intuition," he writes, "is only made possible by pure intuition—that of space and time. Hence what geometry says of the latter will indisputably apply to the former. Such evasions as the statement that objects of sense do not conform to the rules of construction in space (to the principle of the infinite divisibility of lines and angles, for example) must fall to the ground. For such evasions deny to space, and with space to mathematics as a whole, objective validity ; and one no longer knows why and to what extent the mathematics can be applied to phenomena."

Here we have the very nerve of the dispute. Are we to repudiate mathematical reasonings, or, what seems as bad, to deny their applicability to the things of which the senses give us information ? Surely not. But are we, then, to accept the infinite divisibility of what is given in intuition, and must we, to avoid giving offense to the mathematician, shut our eyes and bolt the inevitable consequences of such an admission ? It is pathetic to hear those who feel within them the pangs of the antinomial colic murmur with resignation : "There are, indeed, difficulties," etc.

It is a relief to find that we are not, in fact, shut up to these alternatives. Kant himself has recognized a distinction which, when its significance is clearly seen, enables us to avoid disaster in either direction. The passage in the *Critique*, which I have in mind in saying this, is so interesting that I shall quote it at length:[1]

"We are accustomed to distinguish in phenomena what belongs essentially to the intuition of them, and is valid for every human sense-faculty, from what belongs to them only accidentally, inasmuch as it is not valid in relation to the faculty of sense taken generally, but only in relation to a particular disposition or organization of this or that sense. Knowledge of the first sort gives us, we say, the object as it is in itself; knowledge of the second gives us only the object as it appears. But this distinction is merely empirical. If we adhere to this position (as is commonly done), and do not regard the former empirical intuition (as one should) as, in its turn, mere phenomenon, in which nothing that belongs to the thing-in-itself is to be found, we lose our transcendental distinction, and we believe that we are cognizing things in themselves; whereas, on the contrary, everywhere in in the world of sense, even in our profoundest investigations into the objects which belong to that world, we are dealing with nothing but phenomena.

Thus we call the rainbow a mere appearance or phenomenon in a sunny shower, and we call the rain the thing-in-itself. This is right enough, if we take those words in a mere physical sense, and mean by the thing-in-itself that which, in universal experience, and in all its various relations to the senses, is constituted in intuition in just this way and in no other. But if we take this empirical experience generally, and, without enquiring into its harmony with the faculty of sense of every human being, ask whether this represents an object in itself (not the rain-drops, for they, as phenomena, are evidently empirical objects)—if we do this, we find that the question of the relation of the representative to its object is a transcendental one, and that not only are the drops mere phenomena, but even their globular form, nay, the very space through which they fall, all are nothing in themselves, but are mere modifications or fundamental dispositions of our sensuous intuition. The transcendental object remains unknown to us."

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[1] *Critique of Pure Reason*, General Remarks on Transcendental Æsthetic.

This "transcendental object" is, of course, the "external reality" which has so often been assumed to exist beyond consciousness, and with which I am not concerned in these papers. In this passage of the *Critique*, as in many others, Kant comes near to repudiating it altogether. He sees that the distinction we all draw between appearance and reality does not necessitate any reference to such a thing as this, but is a distinction within our experience, and has to do only with phenomena, in the broad sense of that word. One experience (the rainbow) is taken as the sign of another (the falling drops); the sign is recognized as appearance, while the thing signified takes on the dignity of the reality. This is quite in harmony with the doctrine coming to be accepted, I think, by an increasing number of philosophers, namely, that when we are contrasting in our experience appearance and reality, the reality always means to us that upon which we lay the duty of ordering and explaining our experiences as a whole.

Unhappily Kant did not see the full significance of this distinction. He might, after showing in what sense the rainbow is not the reality, but only the sign of it, have gone on to show that each rain-drop, as visual-appearance, is sign of a reality known to us in terms of touch and motion. Having arrived at this point, he might have indicated that this reality, in its turn, is relatively and not absolutely real; *i. e.*, that what is actually given in sense or imagination (the intuition) may in its turn become sign or appearance of something else, which thus becomes, relatively to it, the reality. As it is, he assumes that there is given in intuition a last 'appearance,' which is the reality, not in a relative, but in an absolute and final sense, and to which the "rules of construction in space" directly apply in all their rigor. He fails to see that here, as before, he is dealing with a symbol, and out of his confusion of symbol and thing symbolized spring the difficulties exhibited above.

The doctrine which I have called the Berkeleian avoids these difficulties, without, I think, giving up anything that the Kantian need care to retain. It merely distinguishes more carefully between symbol and thing symbolized, and refuses to be led into needless perplexities by the assumption of 'necessary forms' of intuition and supposed inferences from them. Its argument may be set forth briefly as follows :—

1. In a given experience of which I am intuitively conscious —say, an expanse of color-sensation—I can distinguish between 'matter' and 'form,' between the stuff of my experience and its arrangement.

2. I perceive the expanse of color to be composite, and to be divisible into parts, but I do not perceive it to be composed of an infinite number of parts, *i. e.*, to be infinitely divisible; so much Kant has himself admitted.

3. It is important to bear in mind, however, that no such single experience constitutes what we mean by a 'real thing,' nor is its 'form' what we mean by 'real space.' We have here only the raw materials out of which real things and real space are built up. Our experiences fall together into an orderly system, and single experiences serve as signs of other experiences or of whole groups of such. Thus the little patch of color sensation that represents a tree seen at a distance, and the larger patch that represents a tree seen near at hand, are recognized as belonging to the same group, and are regarded as different experiences of the *same thing*, *i. e.*, the one can stand for the other, and each serves as a sign of the 'tactual' tree in which the mind rests as the real thing of which each is an appearance.

4. But a little reflection makes it apparent that it is a mistake to suppose that this real thing, of which the whole series of visual appearances are signs, is a single intuitive experience of any sort. The tactual thing, as it exists in the sense or the imagination, is the temporary resting-place of our thought, not its permanent goal. Science conceives the tree to be made up of atoms and molecules, imperceptible to the sense, and yet really existing and furnishing an explanation of what is given in the sense. Of this 'reality' the tree over which I pass my hand becomes an 'appearance.' And if we are justified in thus passing from what is given in the senses, to what science compels us to accept as furnishing its explanation, a path is opened up to us to which we cannot arbitrarily set a limit. The real thing, in any but a rela-

tive sense, becomes to us a possibility of substitutions according to a definite principle; it is not a single intuitive experience of any sort whatever.

5. If we will hold this clearly in mind, we may avoid antinomial pit-falls without either tilting against mathematics, or shocking the common-sense of mankind by denying that space, and lines and angles in space are infinitely divisible. Berkeley pointed out long ago that we cannot continue to subdivide a given finite line (the line, that is, as given in a single intuition) indefinitely. We soon come to what appears to the sense to be a mere point, and to have no part out of part. He rightly indicated that when we talk of subdividing that which seems to the eye a mere point, we are in imagination substituting for that a line, which is, of course, composed of parts, and we are continuing our subdivision upon this substitute. When we realize that this system of substitutions is typical of our whole experience of the real world, which reveals itself in consciousness as a system of interrelated experiences, we can understand why the infinite divisibility of extended things should be so earnestly insisted upon. The point which appears to result from the subdivision of a line can be approached to the eye, and it is seen as a short line. When a further subdivision has taken place, and no change of position will reveal it as a line, we can place a microscope over it. In all this we conceive ourselves to be dealing with the *same* thing, and so we are, in a very important sense of the word *same*. But it is a very unfortunate error to suppose that any one of the experiences which represents to us the real thing is the same with any other in a quite different sense of the word—to suppose, namely, that they are strictly identical. Unless we happen to be psychologists, we are not concerned with any one of the experiences in itself considered. We are concerned with the real thing, of which any single experience is a mere symbol. It is quite possible for the psychologist to maintain that any single experience is probably ultimately divisible into a limited number of sensational elements not themselves further divisible; and yet to maintain stoutly that the real thing is to be conceived as infinitely divisible. He has only to distinguish carefully symbol from thing symbolized.

6. Thus we see that, although the geometer finds his raw materials in intuition, he uses these raw materials only as his point of departure. If lines and angles were not given in intuition, and if we could not subdivide these in individual experiences, the geometrical refinements which have grown out of such experiences would be impossible. But these refinements have, be it remembered, *grown out of* the experiences; they are not identical with the experiences themselves.

For example, a fine line upon the paper before my eye *seems* to me to have length, but no breadth. I can divide it in such a way that the two resulting portions *seem* to me to be exactly equal to each other. I can form an angle out of two such lines, and can draw a third line in such a way that it *seems* to bisect the angle exactly. But the mathematician informs me that no line can be drawn, by any instrument, which has not breadth as well as length; and that the chances are infinitely against the exact equality of the parts of the divided line and of the divided angle. "The line may seem to you without breadth," he explains, "and the line and the angle may seem exactly bisected; but this is mere seeming. If your senses were more discriminating, you would discover your mistake."

This simply means that, in the series of substitutions we have been considering, the line will not remain a line, but will turn into a surface, and the halves will no longer remain halves, but will be seen to be unequal. The geometer gets his first crude notion of a line and of bisection in just such intuitive experiences as I have mentioned. But he does not rest in the intuition; he turns it into a *conception*. The geometrical line he conceives as one which, under all circumstances, is to remain a line; the geometrical point must not, when narrowly inspected, spread out into a spot; the bisected angle must remain bisected. That lines which appear to be true lines are seen on closer inspection to be narrow surfaces, and that visible points turn into small bits of territory, is matter of constant experience. The geometrical line and point must not do this under any circumstances whatever. They are abstractions, not concrete things.

7. From the above it seems to be clear that real space is neither

a hopeless mystery nor the mother of unavoidable self-contradictions. *Real* space is the 'form' of the real thing, and just as the real thing (in any but a relative sense of the word) is not given in any intuition, so real space (in any but a relative sense) is not given in any intuition. When, in any given instance, I pass in thought from appearance to reality—for example, when I pass from the visual appearance to the tactual thing of which it is the sign—I may regard the 'form' of the latter as more real than that of the former. It is that in which the mind rests for the time being. But, as we have seen, any such thing may, in its turn, become appearance in relation to a reality more ultimate; and we recognize that, however far we may carry our investigations, there is no reason to believe that we shall meet with an absolute limit. Every reality in which we may rest at any time is, thus, a relative reality, and its space is relatively real. The absolute object and its absolute space are not *an* object (intuitive) and *a* space (the 'form' of an intuition), but rather *an indefinite series of substitutions* gathered up and hypostatized into an individual. It is to this absolute object and its absolute space that the mathematical conceptions apply in all their rigor. They apply to these without self-contradiction, because we are here not dealing with an individual experience at all.

And it should be noted that, just as we do not think of the several appearances as so many different objects, but call them manifold appearances of the one object; so we do not regard the 'form' of each appearance, the space it occupies, as a distinct and separate space. When we walk toward the tree which we see at a distance, we recognize that we are conscious of a succession of appearances, and a litttle attention to them reveals the fact that they differ from each other both in 'matter' and in 'form'; in other words, the patch of color of which we are conscious undergoes both qualitative and quantitative changes. Yet we maintain that we have been looking all along at the one tree, and we regard that one tree as occupying one real space, which does not grow larger, but remains always the same. This means that both 'matter' and 'form' in the successive appearances have been reduced to the rank of mere signs of a something beyond them.

So much for the Berkeleian doctrine. As it makes any particular finite line in consciousness to consist of a limited number of simple parts, it is not open to the objection that it makes motion along such a line a wholly inconceivable thing. It does not force upon a moving point the absurd task of exhausting an endless series. The descending series discussed in the last paper results after a limited number of terms in the simple, and there the series is broken, for the simple does not consist of parts. In all this there is, at least, no contradiction. In an earlier work I have discussed the objections commonly brought against it, and at the risk of a little repetition I shall quote what I have there said:[1]

"It may be argued, first, as it often is argued, that it is impossible to conceive of any part of a line as not itself extended and having parts. It may be admitted that the small parts arrived at do not *seem* to have part out of part, that these sub-parts are not observed in them; but still it is said that one who thinks about them cannot but think of them as really having such parts. I ask one who puts forward this objection to look into his own mind and see whether he does not mean by 'thinking about them,' bringing them in imagination nearer to the eye, or by some means substituting for them what can be seen to have part out of part. That one can do this no one would think of denying, but this does not prove the original parts to be extended.

It may be objected again that extension can never be built up out of the non-extended—that if one element of a given kind has, taken alone, no extension at all, two or more such elements together cannot have any extension either. I answer that a straight line has no angularity at all, and yet two straight lines may obviously make an angle; that one man is not in the least a crowd, but that one hundred men may be; that no single tree is a forest, but that many trees together do make a forest; that a uniform expanse of color is in no sense a variegated surface, but that several such together do make a variegated surface. It may be that extension is simply the name we give to several simple sense-elements of a particular kind taken together. One cannot say off-hand that it is not.

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[1] *On Sameness and Identity*, pp. 150–152.

"Should one object, finally, that, if a given line in consciousness be composed of a limited number of indivisible elements of sensation, consciousness ought to distinguish these single elements and testify as to their number ; I answer that what is in consciousness is not necessarily in a clear analytical consciousness, nor well distinguished from other elements. For example, I am at present conscious of a stream of sensations which I connect with the hand that holds my pen. The single elements in this complex I cannot distinguish from each other, nor can I give their number. It does not follow that I am to assume the number to be infinite. Much less should I be impelled to make this assumption, if it necessitated my accepting as true what I see to be flatly self-contradictory, as in the case under discussion. It was because of this vagueness and lack of discrimination in the testimony of consciousness that I said, some distance back, that consciousness *seems* to testify that any finite line in it is composed of simple parts. If the testimony were quite clear, the matter would be settled at once. As it is not quite clear, the matter has to be settled on a deductive basis. The most reasonable solution appears to be the Berkeleian."

Surely the Berkeleian doctrine is preferable to the Kantian, and should replace it. But it is desirable not to overlook the fact that the latter doctrine emphasizes a very important truth — it insists strenuously upon the validity of the application of mathematical reasonings to phenomena. In this it is wholly in the right, for here it is recognizing the system of relations which obtains within our experience as a whole. Its only error — that is, its only fundamental error — lies in supposing that in dealing with any single intuition it is dealing with 'real' space and 'real' things. If the Berkeleian will admit that 'real' space is infinitely divisible (as it may be), and if the Kantian will admit that 'real' space is not given in any intuition (as it certainly is not), there need be no quarrel between them.

We shall now turn our attention to the problem of the nature of time.

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